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Hadron masses in cavity quantum chromodynamics to order α_s^2

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Abstract. The non-divergent diagrams describing two-gluon exchange and annihilation between quarks and antiquarks are calculated in the Feynman gauge, based on quantum chromodynamics in a spherical cavity. Using the experimental N, Δ, Ω, and ρ masses to fit the free parameters of the M.I.T. bag model, the predicted states agree very well with the observed low-lying hadrons. As expected, the two-gluon annihilation graphs lift the degeneracy of the π and η, while the ρ and ω remain degenerate. Diagonalizing the $\eta - \eta'$ subspace Hamiltonian yields a very good value for the mass of the η meson.

1 Introduction

Cavity quantum chromodynamics, i.e. quantum chromodynamics with field operators obeying the linear boundary conditions of the M.I.T. bag model on a static sphere (CQCD) [1–3], is a consistent relativistic quantum field theory on its own. Indeed, CQCD may be expanded perturbatively [4], and the diverging Feynman graphs have been shown to be renormalizable [5–8], e.g. in the $\overline{\text{MS}}$ scheme. Taking into account the quadratic boundary condition of the M.I.T. bag model, the properties of the low-energy hadrons have been calculated successfully to order α_s [1–3].

However, this good agreement has been obtained by neglecting the self-energies of the quarks. More recently, the self-energies have been calculated, but unfortunately they turned out to be quite large [6–8], thus spoiling the good agreement to order α_s. One has therefore argued that the self-energies should perhaps be discarded, because the boundary conditions already account for at least part of them.

Moreover, it has been pointed out that the perturbative expansion of CQCD in terms of a power series in the large strong coupling constant may be ill-defined. However, one can argue that the actual value of α_s is immaterial, as long as the spectrum fits, order by order, the same hadronic states, N, Δ, Ω, and ρ, thus fixing the free parameters of the model. Furthermore, it seems obvious that perturbative expansion in terms of cavity modes, rather than plane waves, is a much better starting point for the description of finite size hadrons.

In spite of all these arguments, it is surprising to note that there was in the recent past little enthusiasm to develop CQCD beyond first order to check whether these expectations are actually fulfilled or not. It is, therefore, the purpose of this paper to clarify this issue, by calculating all non-divergent graphs to order α_s^2 in CQCD that can be written in the form of two-body operators, and to compare the resultant low-lying hadron spectrum with the experimental data. We are concentrating only on the two-body interactions in this paper. A calculation of the nondivergent three-body interactions for massless quarks has been performed showing that they are of much less importance than the two-body interactions to order α_s^2. These interactions leading to a three-body force are not included in the following.

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2 Second order energy shift

Using the symmetric form of the Gell-Mann and Low theorem due to Sucher [9], we can write the energy shift of an eigenstate $|\phi_k\rangle$ of the unperturbed Hamiltonian as

$$E_k - E_k^0 = \lim_{\substack{\eta\to 1\\ \varepsilon\to 0_+}} \frac{i\varepsilon}{2} \frac{\partial \langle \hat{\phi}_k | S_\eta^\varepsilon | \hat{\phi}_k \rangle_c / \partial\eta}{\langle \hat{\phi}_k | S_\eta^\varepsilon | \hat{\phi}_k \rangle_c}, \tag{1}$$

where the subscript c indicates that only connected diagrams are included. The adiabatic S-matrix may be expanded as

$$S_\eta^\varepsilon = 1 + \sum_{n=1}^{\infty} S_\eta^{\varepsilon(n)} \tag{2}$$

with

$$S_\eta^{\varepsilon(n)} = \frac{(i\eta)^n}{n!} \int_{-\infty}^{\infty} dt_1 \cdots \int_{-\infty}^{\infty} dt_n \, T\left[\hat{H}_{\text{int}}^\varepsilon(t_1) \cdots \hat{H}_{\text{int}}^\varepsilon(t_n)\right]. \tag{3}$$

$$\Delta E = \lim_{\varepsilon\to 0_+} 2i\varepsilon \int d^4x_1\, d^4x_2\, d^4x_3\, d^4x_4\, e^{-\varepsilon(|t_1|+|t_2|+|t_3|+|t_4|)} \times \left\langle \hat{\phi}_k \left| T\left[\left(\hat{\bar{\psi}} g \frac{\lambda_a}{2} \hat{A}_a \hat{\psi}\right)_{x_1} \left(\hat{\bar{\psi}} g \frac{\lambda_b}{2} \hat{A}_b \hat{\psi}\right)_{x_2} \left(\hat{\bar{\psi}} g \frac{\lambda_c}{2} \hat{A}_c \hat{\psi}\right)_{x_3} \left(\hat{\bar{\psi}} g \frac{\lambda_d}{2} \hat{A}_d \hat{\psi}\right)_{x_4} \right] \right| \hat{\phi}_k \right\rangle . \tag{5}$$

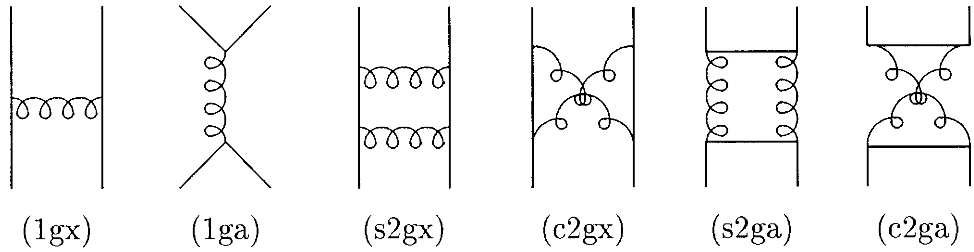

Fig. 1. Finite Feynman diagrams of first and second order in α_s

Here, the parameter $\varepsilon > 0$ introduces the adiabatic switching on of the interaction Hamiltonian $\hat{H}^{\varepsilon}_{\text{int}}(t)$ in the Dirac picture. Expanding ΔE up to second order in α_s, we find that the only non-divergent terms contributing to ΔE are $\langle S^{\varepsilon(2)}\rangle_c$ and $\langle S^{\varepsilon(4)}\rangle_c$, and thus ΔE reads to that order

$$\Delta E = \lim_{\varepsilon\to 0_+} i\varepsilon \left[\langle S^{(2)}\rangle_c + 2\,\langle S^{(4)}\rangle_c \right] . \tag{4}$$

The first term gives rise to the well-known one-gluon exchange and annihilation graphs [4], while the second contains the two-gluon exchange and annihilation graphs which we would like to evaluate in this paper.

Stoddart et al. [10] have calculated the two-body operators in second order including six of the 24 possible time-orderings for both the two-gluon exchange and annihilation graphs. Here we take into account all time-orderings, using the results of [10] as a check. In fact, all the results of [10] can be reproduced with our method.

As an example, let us calculate the 'straight' two-gluon exchange graph (Fig. 1(s2gx)). The energy shift due to this diagram is given by (5) (on top of the page). The time integrals are readily evaluated and the space integrals combine into quark-gluon vertex integrals defined by

$$Q^{m\Sigma}_{nn'} = i \int d^3x\, \bar{u}_n(\mathbf{x})\, \gamma_\mu\, u_{n'}(\mathbf{x})\, a^{\mu}_{m\Sigma}(\mathbf{x}), \tag{6}$$

where $u_n(\mathbf{x})$ and $a^{\mu}_{m\Sigma}(\mathbf{x})$ are the quark and gluon cavity modes, respectively [11]. The result is

$$\Delta E = -g^4 \left\langle \hat{\phi}_k \left| a^{\dagger}_{c'mf} a^{\dagger}_{d'nf'} \left(\frac{\lambda_a}{2}\right)_{c'j} \left(\frac{\lambda_a}{2}\right)_{d'k} \left(\frac{\lambda_b}{2}\right)_{jc} \times \left(\frac{\lambda_b}{2}\right)_{kd} a_{dni'} a_{cmi} \right| \hat{\phi}_k \right\rangle \delta(\varepsilon_i + \varepsilon_{i'} - \varepsilon_f - \varepsilon_{f'}) \times \sum_{\Sigma\Sigma'} \sum_{qq'} \sum_{mm'} \frac{g^{\Sigma\Sigma}\, g^{\Sigma'\Sigma'}}{4\,\Omega^{\Sigma}_m\, \Omega^{\Sigma'}_{m'}}\, \mathcal{Q}_S. \tag{7}$$

Here, we have used the quark and gluon propagators in the Feynman gauge [11]. The subscripts of the quark creation and annihilation operators stand for the color, flavor, and orbital quantum numbers, respectively. Repeated indices on the Gell-Mann matrices indicate a summation according to the Einstein convention. The quark-gluon vertex integrals are combined into the function $\mathcal{Q}_S$ which depends on all the quark and gluon quantum numbers involved in the process,

$$\begin{aligned} \mathcal{Q}_S = {} & Q^{m\Sigma}_{fq}\, Q^{m\Sigma}_{f'q'}\, Q^{m'\Sigma'}_{qi}\, Q^{m'\Sigma'}_{q'i'}\, E_{\text{I}} \\ & - Q^{m\Sigma}_{fq}\, Q^{m\Sigma}_{f'-q'}\, Q^{m'\Sigma'}_{qi}\, Q^{m'\Sigma'}_{-q'i'}\, E_{\text{II}} \\ & - Q^{m\Sigma}_{f-q}\, Q^{m\Sigma}_{f'q'}\, Q^{m'\Sigma'}_{-qi}\, Q^{m'\Sigma'}_{q'i'}\, E_{\text{III}} \\ & + Q^{m\Sigma}_{f-q}\, Q^{m\Sigma}_{f'-q'}\, Q^{m'\Sigma'}_{-qi}\, Q^{m'\Sigma'}_{-q'i'}\, E_{\text{IV}}. \end{aligned} \tag{8}$$

The terms E_{I} to E_{IV}, given in Appendix A, are sums over the energy denominators that arise from the time integrations.

The energy shift can be interpreted as a two-body operator $\hat{V}$ in second quantization sandwiched between the states $|\phi_k\rangle$. This operator can be translated into first quantization, yielding

$$V_{12} = \frac{\alpha_S^2}{R}\, F_{12}^2 \sum_{\Lambda} \nu_{12}(\Lambda), \tag{9}$$

where Λ stands for the total exchanged angular momentum between the quarks. F_{12} denotes the two-body operator $F_{12} = F_1 \cdot F_2$.

Expanding the quark-gluon vertex integrals into angular and radial parts ($S^{m\Sigma}_{nn'}$), we arrive after some algebra at

$$\begin{aligned} \nu_{12}(\Lambda) = {} & -\sum_{\Sigma\Sigma'} \sum_{j_q j_{q'}} \sum_{J_1 J_2} \sum_{\lambda} \frac{g^{\Sigma\Sigma}\eta_\Sigma\, g^{\Sigma'\Sigma'}\eta_{\Sigma'}}{4\,\Omega^{\Sigma}_m R\, \Omega^{\Sigma'}_{m'} R} \\ & \times (-1)^{J_1+J_2+\Lambda} G_{J_1J_2}(f,q,i)\, G_{J_1J_2}(f',q',i')\, \hat{\Lambda}^2 \\ & \times \begin{Bmatrix} j_f & J_1 & j_q \\ J_2 & j_i & \Lambda \end{Bmatrix} \begin{Bmatrix} j_{f'} & J_1 & j_{q'} \\ J_2 & j_{i'} & \Lambda \end{Bmatrix} (-1)^{1+\mu_{f'}-\mu_i} \\ & \times \begin{pmatrix} j_f & \Lambda & j_i \\ -\mu_f & \lambda & \mu_i \end{pmatrix} \begin{pmatrix} j_{f'} & \Lambda & j_{i'} \\ -\mu_{f'} & -\lambda & \mu_{i'} \end{pmatrix} \frac{\mathcal{S}}{R^3} . \end{aligned} \tag{10}$$

Table 1. The numerical results for the coefficients of the two-body operators as defined in (15). The abbreviations stand for one-gluon exchange, straight two-gluon exchange, crossed two-gluon exchange, straight two-gluon annihilation, and crossed two-gluon annihilation, respectively. The second row gives the appropriate color factors

		1gx	**s2gx**	**c2gx**	**s2ga**	**c2ga**
	C_{12}	F_{12}	F_{12}^2	$F_{12}\left(F_{12}+\frac{3}{2}\right)$	$2\left(\frac{16}{27}-\frac{1}{18}F_{12}\right)$	$2\left(-\frac{2}{27}-\frac{5}{9}F_{12}\right)$
$m_1=0$	A	0.009795	−0.406119	0.146124	0.077642	0.286519
$m_2=0$	B	−0.708080	0.747443	−0.190998	0.003926	−0.331505
$m_1=0$	A	0.022668	−0.398787	0.105459	0.020012	0.094710
$m_2=m_s$	B	−0.566556	0.613761	−0.074720	−0.015174	−0.092402
$m_1=m_s$	A	0.052685	−0.422564	0.067182	−0.037616	−0.084483
$m_2=m_s$	B	−0.457013	0.551227	0.009522	−0.017055	0.036234

Here, the summation over the radial quantum numbers of the intermediate quarks and gluons have been omitted for simplicity. The factors G and $\mathcal{S}$ are defined as

$$G_{J_1J_2}(f,q,i)=\hat{\jmath}_f\,\hat{\jmath}_i\,\hat{\jmath}_q^2\,\hat{J}_1\hat{J}_2\,(-1)^{j_f+j_i-j_q} \times\begin{pmatrix} j_f & J_1 & j_q \\ \frac{1}{2} & 0 & -\frac{1}{2}\end{pmatrix}\begin{pmatrix} j_q & J_2 & j_i \\ \frac{1}{2} & 0 & -\frac{1}{2}\end{pmatrix} \tag{11}$$

and

$$\begin{aligned}\mathcal{S}=&\,S_{fq}^{m\Sigma}\,S_{f'q'}^{m\Sigma}\,S_{qi}^{m'\Sigma'}\,S_{q'i'}^{m'\Sigma'}\,E_{\mathrm{I}}\\ &-S_{fq}^{m\Sigma}\,S_{f'-q'}^{m\Sigma}\,S_{qi}^{m'\Sigma'}\,S_{-q'i'}^{m'\Sigma'}\,E_{\mathrm{II}}\\ &-S_{f-q}^{m\Sigma}\,S_{f'q'}^{m\Sigma}\,S_{-qi}^{m'\Sigma'}\,S_{q'i'}^{m'\Sigma'}\,E_{\mathrm{III}}\\ &+S_{f-q}^{m\Sigma}\,S_{f'-q'}^{m\Sigma}\,S_{-qi}^{m'\Sigma'}\,S_{-q'i'}^{m'\Sigma'}\,E_{\mathrm{IV}}\,.\end{aligned} \tag{12}$$

Since we are interested in the interactions of quarks and antiquarks in the ground-state, we can restrict ourselves to $j_n=\frac{1}{2}$ with $n=i,i',f,f'$, for which (10) simplifies to

$$\nu_{12}(0)=-\sum_{\Sigma\Sigma'}\sum_{Jj_qj_{q'}}\frac{g^{\Sigma\Sigma}\eta_\Sigma\,g^{\Sigma'\Sigma'}\eta_{\Sigma'}}{4\,\Omega_m^\Sigma R\,\Omega_{m'}^{\Sigma'}R}\hat{\jmath}_q^2\,\hat{\jmath}_{q'}^2\,\hat{J}^2\,\delta_{J_1J_2} \times\begin{pmatrix}\frac{1}{2} & J & j_q\\ \frac{1}{2} & 0 & -\frac{1}{2}\end{pmatrix}^2\begin{pmatrix}\frac{1}{2} & J & j_{q'}\\ \frac{1}{2} & 0 & -\frac{1}{2}\end{pmatrix}^2\frac{\mathcal{S}}{R^3}, \tag{13}$$

$$\begin{aligned}\nu_{12}(1)=&-8\,S_{12}^2\sum_{\Sigma\Sigma'}\sum_{J_1J_2j_qj_{q'}}\frac{g^{\Sigma\Sigma}\eta_\Sigma\,g^{\Sigma'\Sigma'}\eta_{\Sigma'}}{4\,\Omega_m^\Sigma R\,\Omega_{m'}^{\Sigma'}R}\\ &\times(-1)^{J_1+J_2-j_q-j_{q'}}\,\hat{\jmath}_q^2\,\hat{\jmath}_{q'}^2\,\hat{J}_1^2\,\hat{J}_2^2\\ &\times\begin{Bmatrix}\frac{1}{2} & J_1 & j_q\\ J_2 & \frac{1}{2} & 1\end{Bmatrix}\begin{Bmatrix}\frac{1}{2} & J_1 & j_{q'}\\ J_2 & \frac{1}{2} & 1\end{Bmatrix}\begin{pmatrix}\frac{1}{2} & J_1 & j_q\\ \frac{1}{2} & 0 & -\frac{1}{2}\end{pmatrix}\\ &\times\begin{pmatrix}j_q & J_2 & \frac{1}{2}\\ \frac{1}{2} & 0 & -\frac{1}{2}\end{pmatrix}\begin{pmatrix}\frac{1}{2} & J_1 & j_{q'}\\ \frac{1}{2} & 0 & -\frac{1}{2}\end{pmatrix}\\ &\times\begin{pmatrix}j_{q'} & J_2 & \frac{1}{2}\\ \frac{1}{2} & 0 & -\frac{1}{2}\end{pmatrix}\frac{\mathcal{S}}{R^3}\,.\end{aligned} \tag{14}$$

The ν_{12} can now be determined numerically. We note that our method differs from that of [10] in which the sum over all intermediate states coupled to exterior states with $j=0$ or $j=1$ was calculated, while here we evaluate the coefficients of the two-body operators directly.

3 Results and discussion

The two-body operators for the energy shifts due to the interactions not discussed above may be derived in the same manner. In the following, we take all the two-body interactions into account. The two-body operators of the energy shifts can be written as

$$V_{12}^{(1\mathrm{gx})}=\alpha_\mathrm{s}\,C_{12}^{(1\mathrm{gx})}\left(A^{(1\mathrm{gx})}+B^{(1\mathrm{gx})}\,S_{12}\right) \tag{15a}$$

for the one-gluon exchange graph,

$$V_{12}^{(2\mathrm{gx})}=\alpha_\mathrm{s}^2\,C_{12}^{(2\mathrm{gx})}\left(A^{(2\mathrm{gx})}+B^{(2\mathrm{gx})}\,S_{12}\right) \tag{15b}$$

for the straight and crossed two-gluon exchange graphs, and

$$V_{12}^{(2\mathrm{ga})}=\alpha_\mathrm{s}^2\left(\tfrac{1}{4}-T_{12}\right)C_{12}^{(2\mathrm{ga})}\left(A^{(2\mathrm{ga})}+B^{(2\mathrm{ga})}\,S_{12}\right) \tag{15c}$$

for the straight and crossed two-gluon annihilation graphs. The two-body operators C_{12} and the coefficients A and B are found in Table 1. V_{12} is given in natural units $\hbar c/R$, R being the cavity radius. S_{12}, T_{12} and F_{12} stand for the product of the spin, isospin, and color operators of the two external particles, respectively.

The one-gluon annihilation graph only contributes to the energy shifts of quark-antiquark pairs with the quantum-numbers of a gluon. These do not occur in the hadrons considered here. However, for completeness we quote the resulting two-body operator as well,

$$\begin{aligned}V_{12}^{(1\mathrm{ga})}=&\,0.187505\,\alpha_\mathrm{s}\left(\tfrac{1}{4}-T_{12}\right)\\ &\times\left(F_{12}+\tfrac{4}{3}\right)\left(S_{12}+\tfrac{3}{4}\right).\end{aligned} \tag{16}$$

At first sight, the results for the two-gluon exchange and annihilation seem to disagree with those previously

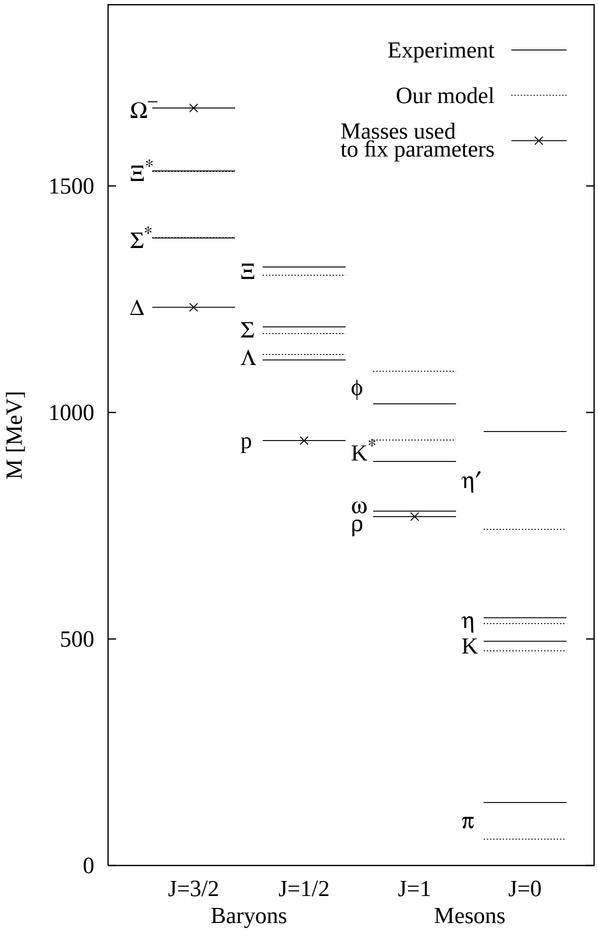

Fig. 2. The hadron spectrum

obtained by Stoddart et al. [10], but since, in contrast to [10], we have included all the time-ordered graphs, this is not surprising. If the coefficients for the two-gluon exchange diagrams are calculated to the same maximal energy including the time-ordered diagrams calculated in [10] only, we are able to reproduce those results. For the two-gluon annihilation graphs, we obtain the same result for the energy-shift of a quark-antiquark pair in a state $j = 0$, even though our coefficients differ from those of [10] for the unphysical case $j = 1$.

Using the mass formula of the M.I.T. bag model [1–3],

$$M = \frac{4}{3}\,(4\pi\,B)^{1/4}\,\Omega^{3/4}, \tag{17}$$

where B is the bag constant and Ω is the total energy of the state in units of $\hbar c/R$, we arrive at the mass spectrum for the low-lying hadrons (Fig. 2). The total energy Ω is of the general form

$$\Omega = N\omega - Z + V_1\alpha_s + V_2\alpha_s^2, \tag{18}$$

where N is the number of quarks, ω the single particle energy, Z the vacuum energy, V_1 and V_2 are the first and second order energy shifts, respectively. Consistent with the philosophy mentioned above, we neglect possible contributions of order α_s and α_s^2 to ω and Z. The masses of the less problematic hadrons, i.e. the nucleon, the Δ-resonance, and the ρ-meson, are used to fix the parameters α_s, B and Z. The Ω^--hyperon fixes the mass of the strange quark. The parameters obtained are

$$\alpha_s = 1.008, \tag{19a}$$
$$Z = 1.471, \tag{19b}$$
$$B^{1/4} = 158.0 \text{ MeV} \tag{19c}$$
$$m_s = 1.445 \text{ fm}^{-1} = 285.1 \text{ MeV}. \tag{19d}$$

The fact that including higher order graphs lowers the value of α_s from 2.2 [12] in first order to just over unity in second order is very encouraging.

The two-gluon annihilation graphs lift the degeneracy of the π and the η mesons while keeping the ρ and ω degenerate. As we are able to calculate the diagonal and off-diagonal matrix elements due to two-gluon annihilation, we may follow the procedure outlined by Donoghue and Gomm [13] and diagonalize the Hamilton matrix, which in the basis $(u\bar{u} + d\bar{d})/\sqrt{2}$ and $s\bar{s}$ reads

$$\hat{\Omega} = \begin{pmatrix} \Omega_{11} & \Omega_{12} \\ \Omega_{21} & \Omega_{22} \end{pmatrix}, \tag{20}$$

with

$$\Omega_{11} = 2\omega_u - Z + \alpha_s\ V_1(u \to u) + \alpha_s^2\ \left(V_{2,e}(u \to u) + 2\,V_{2,a}(u \to u)\right) \tag{21a}$$
$$\Omega_{12} = \sqrt{2}\,\alpha_s^2\ V_{2,a}(u \to s) \tag{21b}$$
$$\Omega_{21} = \sqrt{2}\,\alpha_s^2\ V_{2,a}(s \to u) \tag{21c}$$
$$\Omega_{22} = 2\omega_s - Z + \alpha_s\ V_1(s \to s) + \alpha_s^2\ \left(V_{2,e}(s \to s) + V_{2,a}(s \to s)\right)\ . \tag{21d}$$

Here, u denotes a massless quark, s a strange quark, and the labels e and a refer to interactions due to gluon exchange and annihilation. We differ slightly from the procedure of Donoghue and Gomm [13], as they use the *experimental* values of the pion and kaon masses in their calculation. Deriving the masses of η and η' in the framework of the M.I.T. bag model by diagonalizing the submatrix $\hat{\Omega}$ and inserting the eigenvalues into (17), we obtain

$$m_\eta = 534 \text{ MeV}, \tag{22a}$$
$$m_{\eta'} = 742 \text{ MeV}, \tag{22b}$$

with a mixing angle of 33°. The resulting η mass is very close to the experimental value of $m_\eta = 547$ MeV, while the η' mass is too low compared with $m_{\eta'} = 958$ MeV. However, here we have only taken into account contributions from massless and strange quarks to the mass of the η'. The η' meson could contain a significant $c\bar{c}$ component which might well account for the remaining difference, since heavy quark masses have an important influence on the energy eigenvalues.

As one can see in Fig. 2, the fit is very good for the masses of the baryons. The splitting between the Λ and the Σ is much better than in the calculation to first order in α_s [12]. However, the predictions for the vector mesons ϕ and K^* are still too high, while for the pseudoscalar mesons, π, K, η and η', they are too low.

The fit only takes into account the two-body diagrams calculated in this paper, but none of the divergent graphs to order α_s^2, i.e. those that have a loop on any of the external or internal lines or vertices in Figs. 1(1gx) and 1(1ga). These divergent graphs should be included to make the second order energy shift gauge invariant. However, even though the cavity renormalization techniques have been developed [5–8], the numerical effort required to calculate such graphs is considerable[1] and beyond the scope of this paper. In any case, these contributions are of the same form as the two-body operators for one-gluon exchange and annihilation. They will merely renormalize the first-order contribution by a factor. Thus the part which is orthogonal to the first-order contribution will still be gauge independent. The fit has been found without the introduction of this factor.

4 Conclusion

We have calculated the non-diverging Feynman diagrams up to second order in α_s in the framework of cavity quantum chromodynamics and obtained the two-body operators for the energy shifts. The free parameters of the model, the strong coupling constant α_s, the zero-point energy Z_0, the bag pressure B, and the mass of the strange quark m_s, are fixed by four out of the sixteen available masses of the hadrons containing only up, down, and strange quarks coupled to total angular momentum of up to 3/2. The calculated masses agree very well with the experimental data. It is especially noteworthy, that the mass of the η-meson can be calculated by diagonalizing the η–η' subspace Hamiltonian without introducing any further parameters into the model.

The inclusion of second order interactions leads to a much smaller value of α_s (≈ 1) than only considering first order terms ($\alpha_s > 2$). Self-energies will be included in future work, but at the moment we neglect them as they might be taken care of by the boundary conditions already.

There is still a lot of work ahead, to complete the calculations in cavity QCD to order α_s^2. This includes the evaluation of divergent diagrams, but also the calculation of other observables than the masses, e.g. the magnetic moments, charge radii, and the ratio g_A/g_V.

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[1] The regularization procedure relies on delicate cancellations between oscillating terms.

Appendix A "Straight" two-gluon exchange energy-denominators

$$
\begin{aligned}
E_{\mathrm{I}} = &\Big[(\varepsilon_{q'} - \varepsilon_{i'} + \Omega_{m'}^{\Sigma'})(\varepsilon_{f'} - \varepsilon_{i'} + \Omega_m^{\Sigma} + \Omega_{m'}^{\Sigma'}) \\
&\times(\varepsilon_q - \varepsilon_f + \Omega_m^{\Sigma})\Big]^{-1} + \Big[(\varepsilon_{q'} - \varepsilon_{i'} + \Omega_{m'}^{\Sigma'}) \\
&\times(-\varepsilon_i - \varepsilon_{i'} + \varepsilon_q + \varepsilon_{q'})(\varepsilon_q - \varepsilon_f + \Omega_m^{\Sigma})\Big]^{-1} \\
&+\Big[(\varepsilon_q - \varepsilon_i + \Omega_{m'}^{\Sigma'})(-\varepsilon_i - \varepsilon_{i'} + \varepsilon_q + \varepsilon_{q'}) \\
&\times(\varepsilon_q - \varepsilon_f + \Omega_m^{\Sigma})\Big]^{-1} + \Big[(\varepsilon_{q'} - \varepsilon_{i'} + \Omega_{m'}^{\Sigma'}) \\
&\times(-\varepsilon_i - \varepsilon_{i'} + \varepsilon_q + \varepsilon_{q'})(\varepsilon_{q'} - \varepsilon_{f'} + \Omega_m^{\Sigma})\Big]^{-1} \\
&+\Big[(\varepsilon_q - \varepsilon_i + \Omega_{m'}^{\Sigma'})(-\varepsilon_i - \varepsilon_{i'} + \varepsilon_q + \varepsilon_{q'}) \\
&\times(\varepsilon_{q'} - \varepsilon_{f'} + \Omega_m^{\Sigma})\Big]^{-1} + \Big[(\varepsilon_q - \varepsilon_i + \Omega_{m'}^{\Sigma'}) \\
&\times(\varepsilon_f - \varepsilon_i + \Omega_m^{\Sigma} + \Omega_{m'}^{\Sigma'})(\varepsilon_{q'} - \varepsilon_{f'} + \Omega_m^{\Sigma})\Big]^{-1}
\end{aligned} \quad (A.1)
$$

$$
\begin{aligned}
E_{\mathrm{II}} = &\Big[(\varepsilon_{q'} + \varepsilon_{f'} + \Omega_m^{\Sigma})(\varepsilon_{f'} - \varepsilon_{i'} + \Omega_m^{\Sigma} + \Omega_{m'}^{\Sigma'}) \\
&\times(\varepsilon_q - \varepsilon_f + \Omega_m^{\Sigma})\Big]^{-1} + \Big[(\varepsilon_{q'} + \varepsilon_{f'} + \Omega_m^{\Sigma}) \\
&\times(\varepsilon_{f'} - \varepsilon_i + \varepsilon_q + \varepsilon_{q'} + \Omega_m^{\Sigma} + \Omega_{m'}^{\Sigma'}) \\
&\times(\varepsilon_q - \varepsilon_f + \Omega_m^{\Sigma})\Big]^{-1} + \Big[(\varepsilon_q - \varepsilon_i + \Omega_{m'}^{\Sigma'}) \\
&\times(\varepsilon_{f'} - \varepsilon_i + \varepsilon_q + \varepsilon_{q'} + \Omega_m^{\Sigma} + \Omega_{m'}^{\Sigma'}) \\
&\times(\varepsilon_q - \varepsilon_f + \Omega_m^{\Sigma})\Big]^{-1} + \Big[(\varepsilon_{q'} + \varepsilon_{f'} + \Omega_m^{\Sigma}) \\
&\times(\varepsilon_{f'} - \varepsilon_i + \varepsilon_q + \varepsilon_{q'} + \Omega_m^{\Sigma} + \Omega_{m'}^{\Sigma'}) \\
&\times(\varepsilon_{q'} + \varepsilon_{i'} + \Omega_{m'}^{\Sigma'})\Big]^{-1} + \Big[(\varepsilon_q - \varepsilon_i + \Omega_{m'}^{\Sigma'}) \\
&\times(\varepsilon_{f'} - \varepsilon_i + \varepsilon_q + \varepsilon_{q'} + \Omega_m^{\Sigma} + \Omega_{m'}^{\Sigma'}) \\
&\times(\varepsilon_{q'} + \varepsilon_{i'} + \Omega_{m'}^{\Sigma'})\Big]^{-1} + \Big[(\varepsilon_q - \varepsilon_i + \Omega_{m'}^{\Sigma'}) \\
&\times(\varepsilon_f - \varepsilon_i + \Omega_m^{\Sigma} + \Omega_{m'}^{\Sigma'})(\varepsilon_{q'} + \varepsilon_{i'} + \Omega_{m'}^{\Sigma'})\Big]^{-1}
\end{aligned} \quad (A.2)
$$

$$
\begin{aligned}
E_{\mathrm{III}} = &\Big[(\varepsilon_{q'} - \varepsilon_{i'} + \Omega_{m'}^{\Sigma'})(\varepsilon_{f'} - \varepsilon_{i'} + \Omega_m^{\Sigma} + \Omega_{m'}^{\Sigma'}) \\
&\times(\varepsilon_q + \varepsilon_i + \Omega_{m'}^{\Sigma'})\Big]^{-1} + \Big[(\varepsilon_{q'} - \varepsilon_{i'} + \Omega_{m'}^{\Sigma'}) \\
&\times(\varepsilon_f - \varepsilon_{i'} + \varepsilon_q + \varepsilon_{q'} + \Omega_m^{\Sigma} + \Omega_{m'}^{\Sigma'}) \\
&\times(\varepsilon_q + \varepsilon_i + \Omega_{m'}^{\Sigma'})\Big]^{-1} + \Big[(\varepsilon_{q'} - \varepsilon_{i'} + \Omega_{m'}^{\Sigma'}) \\
&\times(\varepsilon_f - \varepsilon_{i'} + \varepsilon_q + \varepsilon_{q'} + \Omega_m^{\Sigma} + \Omega_{m'}^{\Sigma'}) \\
&\times(\varepsilon_{q'} - \varepsilon_{f'} + \Omega_m^{\Sigma})\Big]^{-1} + \Big[(\varepsilon_q + \varepsilon_f + \Omega_m^{\Sigma}) \\
&\times(\varepsilon_f - \varepsilon_{i'} + \varepsilon_q + \varepsilon_{q'} + \Omega_m^{\Sigma} + \Omega_{m'}^{\Sigma'})
\end{aligned}
$$

$$\times(\varepsilon_q+\varepsilon_i+\Omega_{m'}^{\Sigma'})\Big]^{-1}+\Big[(\varepsilon_q+\varepsilon_f+\Omega_m^{\Sigma})$$
$$\times(\varepsilon_f-\varepsilon_{i'}+\varepsilon_q+\varepsilon_{q'}+\Omega_m^{\Sigma}+\Omega_{m'}^{\Sigma'})$$
$$\times(\varepsilon_{q'}-\varepsilon_{f'}+\Omega_m^{\Sigma})\Big]^{-1}+\Big[(\varepsilon_q+\varepsilon_f+\Omega_m^{\Sigma})$$
$$\times(\varepsilon_f-\varepsilon_i+\Omega_m^{\Sigma}+\Omega_{m'}^{\Sigma'})(\varepsilon_{q'}-\varepsilon_{f'}+\Omega_m^{\Sigma})\Big]^{-1} \quad (A.3)$$

$$E_{\mathrm{IV}}=\Big[(\varepsilon_{q'}+\varepsilon_{f'}+\Omega_m^{\Sigma})(\varepsilon_{f'}-\varepsilon_{i'}+\Omega_m^{\Sigma}+\Omega_{m'}^{\Sigma'})$$
$$\times(\varepsilon_q+\varepsilon_i+\Omega_{m'}^{\Sigma'})\Big]^{-1}+\Big[(\varepsilon_{q'}+\varepsilon_{f'}+\Omega_m^{\Sigma})$$
$$\times(\varepsilon_f+\varepsilon_{f'}+\varepsilon_q+\varepsilon_{q'})(\varepsilon_q+\varepsilon_i+\Omega_{m'}^{\Sigma'})\Big]^{-1}$$
$$+\Big[(\varepsilon_q+\varepsilon_f+\Omega_m^{\Sigma})(\varepsilon_f+\varepsilon_{f'}+\varepsilon_q+\varepsilon_{q'})$$
$$\times(\varepsilon_q+\varepsilon_i+\Omega_{m'}^{\Sigma'})\Big]^{-1}+\Big[(\varepsilon_{q'}+\varepsilon_{f'}+\Omega_m^{\Sigma})$$
$$\times(\varepsilon_f+\varepsilon_{f'}+\varepsilon_q+\varepsilon_{q'})(\varepsilon_{q'}+\varepsilon_{i'}+\Omega_{m'}^{\Sigma'})\Big]^{-1}$$
$$+\Big[(\varepsilon_q+\varepsilon_f+\Omega_m^{\Sigma})(\varepsilon_f+\varepsilon_{f'}+\varepsilon_q+\varepsilon_{q'})$$
$$\times(\varepsilon_{q'}+\varepsilon_{i'}+\Omega_{m'}^{\Sigma'})\Big]^{-1}+\Big[(\varepsilon_q+\varepsilon_f+\Omega_m^{\Sigma})$$
$$\times(\varepsilon_f-\varepsilon_i+\Omega_m^{\Sigma}+\Omega_{m'}^{\Sigma'})(\varepsilon_{q'}+\varepsilon_{i'}+\Omega_{m'}^{\Sigma'})\Big]^{-1} \quad (A.4)$$

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